Who wants to be an auctioneer?

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Motivation

- Most of the literature of decentralized markets typically take trading mechanisms/bargaining protocols as given.
- We endogenize those mechanisms, and the **decision whether to offer a mechanism or to participate in the mechanisms offered by the others**.

- The central questions of our paper:
  - What mechanisms are posted, and how visitors sort among the posted mechanisms?
  - How do traders' costs or valuations affect their posting probabilities?
  - Welfare implications of giving the traders the choice between posting a mechanism and participating in the mechanisms of the others.

Main Features of the Model

- A large number (a continuum) of buyers and sellers.
- Use standard directed/competitive search framework to model matching frictions
  - sellers and buyers post mechanisms visited by the traders on the other side of the market
  - each non-posting trader can only visit one mechanism
  - focus on symmetric strategy equilibrium captures the lack of coordination: each visitor uses the same visiting strategy
- Matching frictions arise from lack of coordination in large markets
  - these frictions cannot be overcome by the planner (or the platform)

Modeling assumptions

- In effect, we extend the basic **competitive search** to study markets with trading protocols set up by **passive (or competitive) platforms**
  - Virag (2019) studies strategic platforms with homogeneous traders
- Similar **questions** can be studied in any **markets not organized as efficient, centralized mechanisms** by the platform(s)
  - the existence of multiple markets with different price clearing mechanisms can be studied even with reduced form pricing mechanisms
  - efficiency of multiple markets, and sorting of traders into different market mechanisms can be studied as well
  - even some centralized mechanisms can be studied but having fully efficient, competitive outcomes in each submarket would trivially lead to a unified, single market or something equivalent

Main results

- **Multiple markets exist**
  - some buyers (sellers) post auctions and some visit a seller's (buyer's) auction
- The equilibrium is constrained **efficient**
  - (constrained efficiency is efficiency subject to matching frictions)
  - market equilibrium maximizes welfare with multiple markets
  - higher welfare than in models with a single market
- **Sellers with lower costs and buyers with higher valuations are more likely to post** when distributions are smooth
  - but if the distributions have large spikes, then monotonicity fails
- Extensive numerical analysis for the two-type model
  - characterized the welfare advantage of segmented markets; **larger heterogeneity in types** implies **a larger welfare gain**
  - monotonicity of posting holds unless inefficient types are predominant

Literature

- McAfee (1993)
- Peters and Severinov (1997)
- Peters (1997)
- Burguet and Sakovics (1999)
- Kultti et al. (2009)
- Ambrus and Argenziano (2009)
- Neeman and Vulkan (2010)
- Eeckhout and Kircher (2010)
- Albrecht, Gautier, Vroman (2014)
- Niedermayer and Shneyerov (2014)
- Jehiel and Lamy (2015)
- Delacroix and Shi (2018)
- Virag (2019)

Model Environment

- A continuum of risk neutral buyers and sellers with mass 1 each
- Each buyer (seller) with a unit demand (unit supply); costs, valuations distributed according to F_c, and F_v on $[0, 1]$ with densities f_c, f_v
- Two potential submarkets
  - In market S, sellers post direct mechanisms; buyers visit sellers' posted mechanisms,
  - In market B, buyers post direct mechanisms; sellers visit buyers' posted mechanisms.
- m_s (m_B) - the mass of sellers (buyers) in market S, rest in market B

Timeline

- Each trader chooses a market, B or S;
  - buyer in market B is a poster, buyer in market S is a visitor
  - seller in market S is a poster, seller in market B is a visitor
- observing mechanisms posted, visitors decide which poster to visit
  - each visitor can only visit one mechanism/poster
- after the visitors make their visiting decisions, the mechanisms are executed

Planner's Problem

- posting probabilities $\beta_s(c), \beta_b(v)$ for each type
- $\tau_v(c)$ the density that a visiting seller with cost c places on visiting buyers with type v
- the best visiting type (seller with lowest c or buyer with highest v) in each match to maximize surplus.

Remarks:

- The functions $\beta_s, \beta_b, \tau_v, \tau_c$ determine the density of the best visiting type at each posting type, this density is denoted by h_c, h_v.
- The planner cannot change the underlying anonymous matching process, hence the term constrained optimum.

Posting probabilities $\beta_s(c), \beta_b(v)$, and visiting densities τ_v, τ_c determine the density of the best visiting type at each posting type, h_c, h_v.
The planner's problem is

$$\max_{\beta_s,\beta_b,\tau_c,\tau_v} \int_0^1 \beta_s(c) \int_c^1 h_c(v)(v-c)dvdc + \int_0^1 \beta_b(v) \int_v^1 h_v(c)(v-c)dvdc.$$

Lemma

The social welfare function is strictly concave, and thus there is a unique welfare optimum.

For our efficiency result, we show that the first order conditions of this problem coincide with the first order conditions of equilibrium conditions.

Posting Equilibrium in a Market

What (direct) mechanism should a seller A with cost c in market S offer?

- Mechanism representation: (q, w) - allocation rule and the utility schedule w for each buyer type.
- $u(v)$ is the utility that the buyer of type v earns in market S by optimally participating there.
  - This is taken as given in our competitive search model.
- IC implies that u is continuous, increasing and convex.
- Sequential Rationality: A buyer type v participates in the mechanism offered by seller A only if $w(v) \geq u(v)$.

- Result: Given the market schedule u, the mechanism offered by seller A is a choice of the allocation rule $q(v)$, and the participation rate (queue length) $\lambda(v)$ at seller A for each buyer type v.

Best reply and equilibrium for the posters' market (for sellers)

Proposition 1. For any utility schedule u such that $u(1) < 1 - c$, the unique optimal queue length schedule, and the unique optimal allocation rule are both induced by a second price auction with a reservation price equal to c.

Proposition 2. In every equilibrium with direct mechanisms, the queue lengths induced and the assignment rule implemented at any seller are identical to the equilibrium where each seller posts a second price auction with a reservation price equal to his cost.

Given these results, from now on we focus on the particularly tractable second-price auction mechanism with the appropriate reservation prices.

Directed search equilibrium for the entire economy: refinement

There are always trivial equilibria where only one market opens due to coordination frictions.

This is easily refined away by assuming that an arbitrarily small fraction of each side (buyers and sellers) goes to each market
or
by assuming that each type who makes a zero utility when posting does visiting with probability one.

Equilibrium and optimum

Proposition 3. Given our refinement, there exists a unique equilibrium tuple $(\beta_s, \beta_b, \tau_v, \tau_c)$. The unique equilibrium decentralizes the unique constrained optimum.

Discussion: The proof relies on showing that the first order conditions for equilibrium and for social optimum are identical and that they satisfy strict concavity.

The result extends the results of the literature that proves such a result when the posting and visiting sides are fixed. We show that also the posting/visiting decisions internalize the externality on the other agents when markets are large.

Equilibrium and optimum: discussion

Proposition 3. There exists a unique equilibrium tuple $(\beta_s, \beta_b, \tau_v, \tau_c)$. The unique equilibrium decentralizes the unique constrained optimum.

The existence of two submarkets

- Allows each type to trade with a positive probability.
- Creates extra trades matching low value buyers with high value sellers where high value sellers post auctions.
- Such a matching would not be possible if sellers were forced to post.
- In that case, trading volumes and welfare would be substantially lower. In particular, if sellers were posting only, then low value sellers would be shut out of the market fully.
- When costs and valuations are both uniform, a full 39% of the sellers would be shut out in the single-market case.

Posting/visiting patterns

A main question is whether more efficient types (lower c or higher v) are more likely to post than less efficient types.
For the sellers, this boils down to whether β_s is decreasing.

- Inefficient types (buyer with low values, sellers with high costs) only visit. This is intuitive, since they cannot attract any visitors if posting.
- More efficient types post with a positive probability
- But the probability of posting is less than 1 for all types, except for the most efficient type.

Theorem

In equilibrium, for all $v < 1$ *and* $c > 0$, $\beta_b(v), \beta_s(c) < 1$. *There exist cutoffs* $\underline{v}, \overline{v}, \underline{c}, \overline{c} \in [0,1]$ *such that*
(a) for all $v \in [0, \underline{v})$, $c \in (\overline{c}, 1]$ *it holds that* $\beta_b(v) = \beta_s(c) = 0$;
(b) for all $v \in [\overline{v}, 1]$, $c \in [0, \underline{c}]$ *it holds that* $\beta_b(v) = \beta_s(c) > 0$.

Posting/visiting patterns cont.

We have further positive and negative results:
1a. If $f_v(0)/f_c(0) \geq 1/2$ then $\beta_s(0) = 1$ and sellers with $c = 0$ post with a probability equal to 1 in equilibrium.
1b. If $f_v(0)/f_c(0) < 1/2$ then $\beta_s(0) = 2f_v(0)/f_c(0) < 1$, and sellers with $c = 0$ post with a probability less than 1 in equilibrium. In this case, monotonicity of β_s at $x = 0$ holds only if f_v/f_c is increasing at $x = 0$.

2. With two types ($v = 1$ and $1 - \alpha$; and $c = 0$ or α) we obtained numerical comparative static results that show that such monotonicity is the norm but does not always hold. It fails when the less efficient type is much more common.

Posting/visiting patterns:further discussion

2. With two types monotonicity fails when the less efficient type is much more common.
3. Conjecture: if f_v is increasing and f_c is decreasing, and $f_v(0)/f_c(0)$ is large enough, then β_s is decreasing in c.
4. Proven: When F_v and F_c are uniform (or close enough to uniform), then monotonicity holds.

In general, referring to 2-4 above, **failure of monotonicity** arises when there is a **high density (**or atom) **of a type**.

- Both types below and above this frequent type prefer to visit over posting because of the extra sensitivity of trading probabilities when visiting.
- This will make indifference between the two markets impossible for those types, and thus monotonicity fails in this case.

Conclusion

- Contributing to the directed search literature, this paper endogenizes the agents' roles in the economy by **allowing a trader on each side of the market either to post a mechanism or respond/visit other mechanisms**.
- The directed search equilibrium is **constrained efficient**.
- With bilateral posting and visiting, market **welfare is significantly higher than in the market with one-sided posting**.
- **Larger heterogeneity in types implies a larger welfare gain**

Conclusion

- We allow **rich heterogeneity** in the population and demonstrate that **the choice between posting and visiting depends on the trader's type** (value or cost).
- Less efficient types only visit. More efficient types post with a positive probability.
- **Under a range of distributions**, including the **uniform distribution, posting probabilities are monotone in types**.