

# Dynamic Privacy Choices

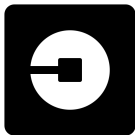
Shota Ichihashi

Bank of Canada

CETC 2020

The views expressed are those of the author and do not necessarily reflect the views of the Bank of Canada.

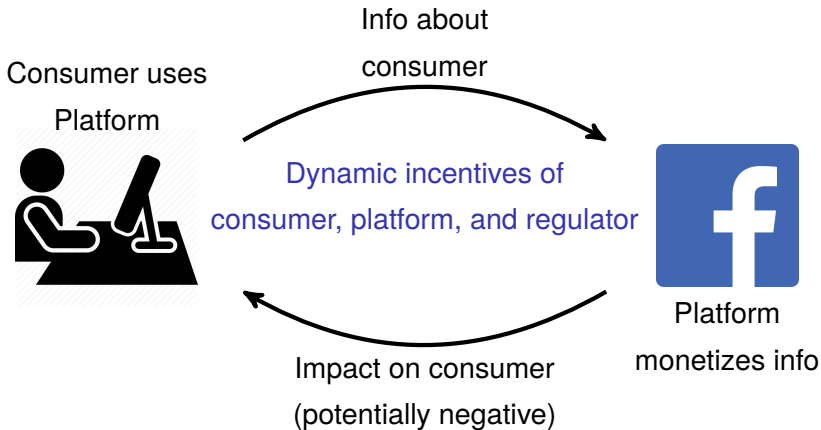
# Motivation



amazon

Baidu 百度

# This Paper



# Roadmap

## 1. Monopoly

- ▶ Model
- ▶ Equilibrium

## 2. Competition

- ▶ Model
- ▶ Equilibrium

# Model

Time  $t = 1, 2, \dots$

Consumer

- ▶ Choose an activity level  $a_t \geq 0$
- ▶ Type  $X \sim \mathcal{N}(0, \sigma_0^2)$ , fixed over time, unobservable<sup>1</sup>

Platform

- ▶ Privately observe  $s_t = X + \varepsilon_t$  with  $\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)$
- ▶  $\gamma_t$ : privacy level in period  $t$

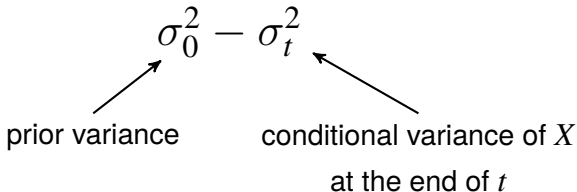
All random variables are mutually independent

---

<sup>1</sup>If privately observable, focus on a “pooling” equilib.

# Platform Payoffs

Platform's payoff in period  $t$



- ▶ More info better
- ▶ Increasing in  $(a_1, \dots, a_t)$  and decreasing in  $(\gamma_1, \dots, \gamma_t)$
- ▶ Take  $\sigma_t^2$  as a primitive (depends on  $a_1, \dots, a_t$  and  $\gamma_1, \dots, \gamma_t$ )
- ▶ Discount future payoffs

# Consumer Payoffs

Consumer payoff in period  $t$

$$u(a_t) - \overbrace{v \cdot (\sigma_0^2 - \sigma_t^2)}^{\text{privacy cost}}$$

$u(\cdot)$  strictly concave,  
maximized at  $a^* > 0$

value of privacy

Consumer myopically chooses  $a_1, a_2, \dots$

# Timing

1. Platform chooses a *privacy policy*  $(\gamma_1, \gamma_2, \dots) \in \mathbb{R}_+^\infty$

- ▶  $s_t = X + \varepsilon_t$  with  $\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)$

- ▶ Higher  $\gamma_t \rightarrow$  collect less info (for a fixed  $a_t$ )

2. Consumer (myopically) chooses  $a_1, a_2, \dots$

Equilibrium: Consumer myopic best response + Platform optimality



# Roadmap

## 1. Monopoly

- ▶ Model
- ▶ Equilibrium

## 2. Competition

- ▶ Model
- ▶ Equilibrium

# Timing

1. Platform chooses a privacy policy  $(\gamma_1, \gamma_2, \dots)$

▶  $s_t = X + \varepsilon_t$  with  $\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)$

2. Consumer (myopically) chooses  $a_1, a_2, \dots$

## Static Problem

Since myopic, Consumer chooses  $a_t$  to maximize

$$\begin{aligned} & u(a_t) - v(\sigma_0^2 - \sigma_t^2) \\ &= u(a_t) - v \left( \underbrace{\sigma_0^2 - \frac{1}{\frac{1}{\sigma_{t-1}^2} + \frac{1}{a_t + \gamma_t}}}_{= C_t} \right) \end{aligned}$$

# Marginal Privacy Cost

## Lemma

$\frac{\partial C_t}{\partial a_t}$  is increasing in  $\sigma_{t-1}^2$ .

- ▶ Less privacy (lower  $\sigma_{t-1}^2$ ) → Lower **marginal** cost
- ▶ E.g. if platform knows  $X$ , zero marginal cost
- ▶ **Concave** cost of producing info
- ▶ Lower payoff  $\leftrightarrow$  Higher incentive to use the same platform

# Timing

1. Platform chooses a privacy policy  $(\gamma_1, \gamma_2, \dots)$

▶  $s_t = X + \varepsilon_t$  with  $\varepsilon_t \sim \mathcal{N}(0, \frac{1}{a_t} + \gamma_t)$

2. Consumer (myopically) chooses  $a_1, a_2, \dots$

# Timing

1. Platform chooses a privacy policy  $(\gamma, \gamma, \dots)$

▶  $s_t = X + \varepsilon_t$  with  $\varepsilon_t \sim \mathcal{N}(0, \frac{1}{a_t} + \gamma)$

2. Consumer (myopically) chooses  $a_1, a_2, \dots$

# Stationary Privacy Policy

Reminder:  $u(a_t) - v \cdot (\sigma_0^2 - \sigma_t^2)$ ,  $a^* := \arg \max_{a \geq 0} u(a)$

## Proposition

Suppose  $\gamma_t = \gamma$  for all  $t$ . There is a  $v^*(\gamma) > 0$  such that

1.  $v > v^*(\gamma) \Rightarrow a_t^* = 0$  for all  $t$ .
2.  $v < v^*(\gamma) \Rightarrow a_t^*$  is increasing and converges to  $a^*$ , and  $\sigma_t^2 \rightarrow 0$ .

Moreover,  $v^*(\gamma)$  is increasing in  $\gamma$ .

- ▶  $a_1 > 0 \rightarrow$  lower marginal cost in  $t = 2 \rightarrow a_2 \geq a_1 \dots$
- ▶ High  $\gamma$  may **lower** the long-run welfare of myopic consumers

# Equilibrium

1. Platform chooses a privacy policy  $(\gamma_1, \gamma_2, \dots)$
2. Consumer (myopically) chooses  $a_1, a_2, \dots$



# Equilibrium

Reminder:  $u(a) - v \cdot (\sigma_0^2 - \sigma_t^2)$ ,  $a^* = \arg \max_{a \geq 0} u(a)$

## Proposition

For any  $v \in \mathbb{R}$ , in any equilibrium, the following holds.

1.  $\sigma_t^2 \rightarrow 0$  and  $a_t^* \rightarrow a^*$
2.  $\forall \tau \in \mathbb{N}$ ,  $\exists v^* > 0$  s.t.  $\forall v \geq v^*$ ,  $\gamma_t^* > 0$  for  $t = 1, \dots, \tau$
3.  $\gamma_t^* \rightarrow 0$ 
  - ▶ Early : high marginal cost  $\rightarrow$  high  $\gamma_t$  to encourage activity
  - ▶ Later : low marginal cost  $\rightarrow$  low  $\gamma_t$  to speed up learning
  - ▶ Point 1 holds for, e.g., a patient consumer or heterogeneous  $v$

# Implication

Two conditions under which

- ▶ consumer privacy is difficult to sustain
- ▶ collecting and monetizing consumer data is effective

1. Declining marginal privacy cost (concave loss of providing info)

- ▶ Rational addiction: Becker and Murphy (1988)
- ▶ “Privacy paradox”

2. Platform’s ability to commit to “underuse” data

# Equilibrium Privacy Policy (characterized in paper)

Platform's trade-off

- ▶ Higher  $\gamma_t$  increases activity but reduces the precision of signal

Platform's optimization = static optimization

(set  $\gamma_t$  to maximize the period- $t$  profit)

$$\text{Platform objective} = \underbrace{\text{today's profit}}_{\text{increasing in today's info}} + \underbrace{\text{continuation value}}_{\text{increasing in today's info}}$$

One-period commitment  $\rightarrow$  full commitment outcome

# Roadmap

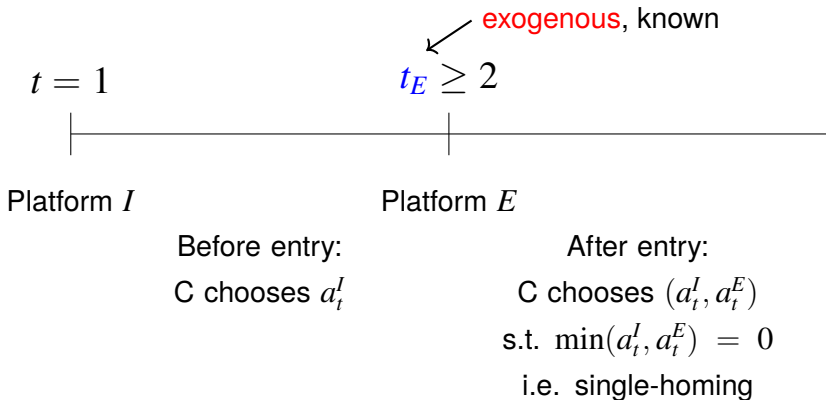
## 1. Monopoly

- ▶ High  $a_t$ , privacy loss, potentially negative payoff in the long-run
- ▶ Strong commitment power unnecessary
- ▶ Less privacy  $\rightarrow$  lower incentive to protect it

## 2. Competition

- ▶ Model
- ▶ Equilibrium

# Incumbent and Entrant



# Payoffs

- ▶ Platform  $k$ :  $\sigma_0^2 - \sigma_{k,t}^2$
- ▶ Consumer before entry:  $u(a_t^I) - v \cdot (\sigma_0^2 - \sigma_{I,t}^2)$
- ▶ Consumer after entry

$$u(a_t^I) - v \cdot (\sigma_0^2 - \sigma_{I,t}^2) + u(a_t^E) - v \cdot (\sigma_0^2 - \sigma_{E,t}^2)$$

- ▶ Privacy cost does not disappear after switching!  
e.g. technical problem, credibility
- ▶ Consumer decision based on **marginal** cost

# Roadmap

## 1. Monopoly

- ▶ High  $a_t$ , privacy loss, potentially negative payoff in the long-run
- ▶ Strong commitment power unnecessary
- ▶ Less privacy  $\rightarrow$  lower incentive to protect it

## 2. Competition

- ▶ Model
- ▶ **Equilibrium**

# Equilibrium

## Proposition

*There is an equilibrium such that:*

- ▶ *Consumer only uses I:  $a_t^I > 0$  and  $a_t^E = 0$  for all  $t$ .*
- ▶  *$\gamma_t^I \rightarrow 0$ ,  $\sigma_{I,t}^2 \rightarrow 0$ , and  $a_t^I \rightarrow a^*$ .*

*For a large  $t_E$ , in any equilibrium, I chooses a monopoly strategy.*



# Intuition

- ▶ Incumbent obtains data
- ▶ Lower marginal privacy cost → cheaper to keep using  $I$
- ▶ Switching / entry less likely
- ▶ Data as an incumbency advantage
- ▶ Market for search engines, Google vs. DuckDuckGo?
- ▶ Right to be forgotten (enabling consumer to erase past info)

## Literature (not exhaustive!)

Consumer data : Choi et al. (2018), Easley et al. (2018), Acemoglu et al. (2019), Bergemann et al. (2019), Bonatti and Cisternas (2020), Frankel and Kartik (2020), Liang and Madsen (2020)

Competition with data: Cornière and Taylor (2020); Prufer and Schottmüller (2017); Hagiu and Wright (2020)

Switching cost, barrier to entry: Farrell and Shapiro (1988); Klemperer (1995); Fudenberg and Tirole (2000)

Signal-jamming: Holmstrom (1999), Horner and Lambert (2019)

# Recap

Less privacy  $\Rightarrow$  lower marginal incentive to protect privacy

## Monopoly

- ▶ Long-run: High activity level & low privacy
- ▶ High privacy level only in early periods

## Competition

- ▶ Lower marginal cost prevents successful entry
- ▶ Entry less likely when consumer welfare is low

## Regulation

- ▶ “Right to be forgotten” might be effective

# Total Welfare

Difficult to compare the sum of payoffs

- ▶ Different discount factors

For any discount factors, we can calculate long-run welfare

- ▶ Without transfer  $\rightarrow$  eqm efficient (platform best)
- ▶ With transfer  $\rightarrow$  efficient iff  $\nu < 1$

Same  $\delta$  & transfer & compare discounted total surplus

- ▶ Inefficiently low activity levels?
- ▶ Inefficiently low privacy level?