How To Sell in a Sequential Auction Market

Ken Hendricks[1] Thomas Wiseman[2]

[1]Department of Economics
University of Wisconsin, Madison

[2]Department of Economics
University of Texas at Austin

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Summary

QUESTION: When a seller faces competition from a subsequent auction, what mechanism maximizes expected revenue?

ANSWER: Not a standard auction with a reserve price.

- The optimal reserve rule depends on second- and third-highest valuations.
    - Allocation to second-highest bidder.
- Implementable through auction where top 2 bidders pay.
- Does much better than standard auction.
- First-order approach fails, so solving requires new method.

Outline

Outline

Model

- 2 sellers.
- $N \geq 3$ buyers.
- After first mechanism runs, second-price auction among remaining buyers for unit 2.

Model

- 2 sellers
  - Each has 1 identical unit of a good.
  - No cost, no value to sellers.
- $N \geq 3$ buyers.
  - Unit demand.
  - Private i.i.d. valuations $v_i \sim F$.
  - $v_{(k)}$ denotes the k-th highest realized valuation.
- (For today, $N = 3$ and F is $U[0,1]$.)
- After first mechanism runs, second-price auction among remaining buyers for unit 2.
  - **Baseline**: second seller is non-strategic.
  - **Baseline**: no reserve price in second auction.

Our Questions

- What mechanism maximizes the (first) seller's expected revenue?
  - Potential future interaction creates complications – externalities, common value.
  - Lots of things are sold this way.
- What is the result of sequential competition in mechanisms?
  - Existing literature on competing simultaneous mechanisms.
    - Burguet and Sakovics (1999): competition in reserve prices.
    - McAfee (1993), Peters and Severinov (1997), Pai (2009): more general mechanisms.

Outline

Our Answer: optimal mechanism when $r_2 = 0$

Theorem

Optimal rule is to allocate, to second-highest buyer, iff

$$\psi\left(v_{(2)}\right) + v_{(2)} - v_{(3)} \geq 0.$$

- $\psi(v) \equiv v - \frac{1-F(v)}{f(v)}$. **(Virtual valuation)**

Intuition: standard, one-seller, one-item case

- Total surplus is $v_{(1)}$.
- IC limits seller to $\psi\left(v_{(1)}\right)$.
- Optimal rule: allocate iff $\psi\left(v_{(1)}\right) \geq 0$.
  - Implementable through first- or second-price auction with reserve price $r^* = \psi^{-1}(0)$.
  - Uniform example: $\psi(v) = 2v - 1$, so $r^* = \frac{1}{2}$.

Intuition for optimal mechanism in our setting

- What surplus does seller 1 create in our setting?
- In absence of seller 1, total surplus for buyers is $v_{(1)} - v_{(2)}$: outcome from second auction.
    - If seller 1 allocates to highest type, surplus is $v_{(1)} + \left[v_{(2)} - v_{(3)}\right]$.
    - If seller 1 allocates to second-highest type, surplus is $v_{(2)} + \left[v_{(1)} - v_{(3)}\right]$.
    - Either way, increase is $v_{(2)} + v_{(2)} - v_{(3)}$.
- IC: seller 1 can get $\psi\left(v_{(2)}\right) + v_{(2)} - v_{(3)}$.
- Optimal rule: allocate iff $\psi\left(v_{(2)}\right) + v_{(2)} - v_{(3)} \geq 0$.
    - always allocate if $\psi\left(v_{(2)}\right) \geq 0$.
    - may allocate even if $\psi\left(v_{(2)}\right) < 0$, because $v_{(2)} - v_{(3)} \geq 0$ also contributes to surplus.

Another way to see it: gross payoff to a bidder

p^k: prob. that mechanism allocates to k-th highest bidder.

- Bidder (1) with valuation $v_{(1)}$ gets

$$p^1 \cdot v_{(1)} + p^2 \cdot \left[v_{(1)} - v_{(3)}\right] + \left(1 - p^1 - p^2\right) \cdot \left[v_{(1)} - v_{(2)}\right]$$
$$=$$
$$\left[v_{(1)} - v_{(2)}\right] + p^1 \cdot v_{(2)} + p^2 \cdot \left[v_{(2)} - v_{(3)}\right].$$

- Bidder (2) with valuation $v_{(2)}$ gets

$$p^2 \cdot v_{(2)} + p^1 \cdot \left[v_{(2)} - v_{(3)}\right].$$

- Bidder (3) with valuation $v_{(3)}$ gets

$$p^3 \cdot v_{(3)}.$$

Benefits of allocating

- Bidder (1) with valuation $v_{(1)}$ gets

$$\left[v_{(1)} - v_{(2)}\right] + p^1 \cdot v_{(2)} + p^2 \cdot \left[v_{(2)} - v_{(3)}\right].$$

- Bidder (2) with valuation $v_{(2)}$ gets

$$p^2 \cdot v_{(2)} + p^1 \cdot \left[v_{(2)} - v_{(3)}\right].$$

- Effect of increasing p^1:
  - Bidder (2) gains $v_{(2)} - v_{(3)}$;
  - Bidder (1) gains $v_{(2)}$.
- Effect of increasing p^2:
  - Bidder (1) gains $v_{(2)} - v_{(3)}$;
  - Bidder (2) gains $v_{(2)}$.

Allocate to whom?

- Seller 1's optimal mechanism given first-order IC specifies allocation to *either* of top two bidders.
- Allocating to Bidder (2) satisfies global IC, but allocating to Bidder (1) may not.
- So:

Theorem

Optimal rule is to allocate, to second-highest buyer, iff

$$\psi\left(v_{(2)}\right) + v_{(2)} - v_{(3)} \geq 0.$$

Uniform example

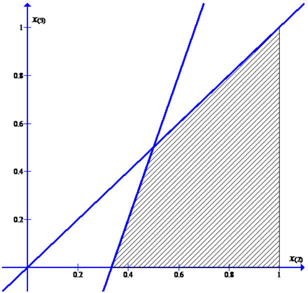
- Rule is to allocate if

$$\psi\left(v_{(2)}\right) + v_{(2)} - v_{(3)} \geq 0.$$

- In uniform example, $\psi(v) = 2v - 1$, so allocate if

$$3v_{(2)} - 1 - v_{(3)} \geq 0.$$

Allocation region for uniform example



- **Allocate if** $3v_{(2)} - 1 - v_{(3)} \geq 0$.
  - Always if $v_{(2)} \geq \frac{1}{2}$ (standard optimal reserve price);
  - Never if $v_{(2)} < \frac{1}{3}$;
  - Sometimes if $v_{(2)} \in \left(\frac{1}{3}, \frac{1}{2}\right)$.

Outline

Implementing the optimal mechanism

- Modified third-price auction.
  - payments from highest and second-highest bidders if item is allocated.
  - bidding valuation is an ex post equilibrium.
- Modified pay-your-bid auction with a rebate.
  - second-highest bidder pays if item is allocated.
  - highest bidder pays unconditionally, and gets a rebate equal to price he pays in second auction.
  - equilibrium bid function strictly increasing.
- Optimal revenue is $\frac{55}{144} \approx 0.382$.

Comparison to second-price, no-reserve auction

- A second-price, no-reserve auction yields expected revenue $\mathbb{E}V_{(3)} = \frac{1}{4}$.
  - Same expected price in both auctions.
  - Can show that this is optimal if Seller 1 must allocate his item.
- We saw that Seller 1's threat to withhold can increase expected revenue.
  - A reserve price is another way to withhold ...

Standard auction with reserve price does badly

Theorem

With any non-trivial reserve price r_1, there is no strictly increasing, symmetric equilibrium of either a first-price auction or a second-price auction for the first item.

- Jehiel and Moldovanu (2003): non-existence in second-price auction with positive externalities.
- Second-price auction has symmetric equilibrium with partial pooling at the reserve price.
  - can calculate optimal $r_1^* \approx 0.379$.
  - lower revenue.
    - lots of non-participation.
    - item may go to third-highest bidder.

Revenue comparisons: $U[0,1], N = 3$

Revenue Comparisons		
	Seller 1's Revenue	Seller 2's Revenue
Optimal Mechanism	0.382	0.289
Must-Sell Mechanism	0.250	0.250
Optimal Second-Price Auction	0.303	0.282

- Optimal mechanism increases revenue for both sellers.
  - 54% for seller 1.
- Standard auction gives seller 1 only 40% of optimal increase.

Outline

1. Model
2. Optimal mechanism when $r_2 = 0$
3. Implementation and revenue comparisons
4. Further results
5. Summary and discussion

Finding the optimal mechanism when $r_2 > 0$

- Suppose that Seller 2 has a non-trivial reserve price $r_2 > 0$.
- We get qualitatively similar results.
- New effect: when $r_2 > 0$, Seller 1's optimal mechanism given first-order IC does not satisfy global IC.
    - requires new approach to solve.
- Allocation rule now may depend on highest value $v_{(1)}$ also.

Making Seller 2 strategic

- Now suppose that Seller 2 chooses reserve price r_2 knowing that Seller 1 will respond optimally.
- Again, qualitatively similar results.
- Seller 2's equilibrium choice of $r_2 > 0$ lowers Seller 1's maximized revenue in example.
  - not obvious: less competition for Seller 1 but also less surplus to divide.

Competing mechanisms

- Seller 2 chooses r_2, Seller 1 best responds as above.
- For Seller 2, marginal change in r_2 has two effects:
  - usual tradeoff b/w higher price and lower prob. of sale;
  - also affects Seller 1's mechanism.
    - lower probability that Seller 1 allocates benefits Seller 2.

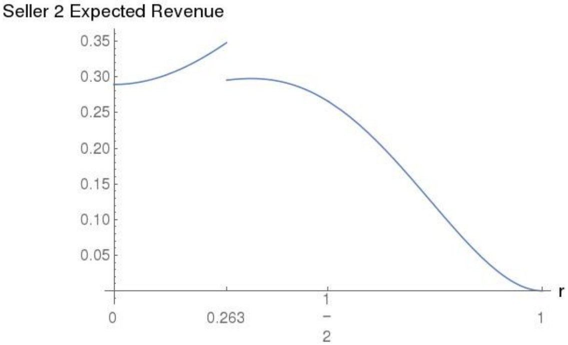


Figure: Seller 2's revenue as a function of r_2

$U[0,1], N = 3$ example

Theorem

Equilibrium in uniform example: $r_2^* \approx 0.263$.

- Relative to $r_2 = 0$, Seller 2's revenue increases from 0.289 to 0.341.
- Seller 1's revenue falls from 0.382 to 0.343.
  - less competition $\Rightarrow$ easier for Seller 1 to extract surplus,
  - but less surplus to extract.
  - second effect dominates here.
- In general, the effect of r_2 on Seller 1's revenue is non-monotonic.
  - at $r_2 \geq 1$ (no Seller 2), Seller 1's revenue is 0.531.

Outline

Summary

- Characterize the optimal mechanism when a seller faces competition from a subsequent auction.
  - allocation rule depends on $v_{(2)}$, $v_{(3)}$, and sometimes $v_{(1)}$.
- Implementation through third-price auction or pay-your-bid auction with rebate.
  - but standard auction not very effective.
- Characterize outcome of competition in mechanisms.
- Technical contribution in solving mechanism design problem where first-order approach fails to satisfy global IC.
  - Carroll and Segal (2019) and Bergemann, Brooks, and Morris (2019) face similar failures.
    - resale introduces externalities, common values.

Interesting questions

- Extension to more items per seller is easy.
- Extension to more than 2 sellers is harder.
  - information leakage.
  - maybe restrict later sellers to EPIC mechanisms?
- This paper is a first step toward studying sequential competition between sellers.
  - Peters (2010) argues that competition among sellers promotes simple, more efficient mechanisms.
  - Our results suggest that that conclusion may not hold when auctions are sequenced.