Viral Social Learning

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Motivation: idea/product that spreads like a virus

Example 1: job-market papers

- Hiring committees learn about JMPs socially, as well as in applications
- For a committee member: not knowing a paper prior to interview season is a bad signal
- For a candidate: when to publish JMP matters
Motivation: idea/product that spreads like a virus

Example 2: influencer marketing
- Customers find out new products from others’ adoption
- If a better product is adopted faster, seeing it early is a good signal
- Viral campaign vs. advertising campaign for producer
This paper

Questions

- Demand side: lifecycle of viral diffusion?
- Supply side: is a viral campaign optimal? For how long? Does it facilitate quality improvement?

Novelty of problem

- Endogeneity of awareness and action sequence
- Time-varying inference affected by producer’s strategy
Classical models of social learning

- **Foundation**: Bikhchandani et al. (1992), Banerjee (1992), Smith and Sorensen (2000)

SIR model of viral transmission

- **Theory**: Kermack and McKendrick (1927, 1932, 1933)
- **Applications**: Newman (2002), McAdams (2017a,b)

Other works on viral marketing and competitive contagion

- Kempe et al. (2003, 2005), Mossel and Roch (2010)
- Goyal et al. (2014)
Basic structure

One product with uncertain quality
- Quality $\omega \in \{g, b\}$, with probability $\alpha \in (0, 1)$ and $1 - \alpha$
- Sells at price $p \geq 0$
- Good product has value $\bar{u}$, bad product $u$, assume $p = \frac{\bar{u} + u}{2}$

A unit mass of consumers
- Each has a unit demand
- At $t = 0$ of a continuous time line, a small $\Delta > 0$ fraction of consumers are exposed to the product

Independent private signals
- Consumer $i$ sees $s_i \in \{G, B\}$ upon exposure
- $\text{Prob}(s_i = G|g) = \text{Prob}(s_i = B|b) = \rho \in (\frac{1}{2}, 1)$
Viral transmission

Three types of consumers

- Susceptible: $S_\omega(t)$. $S_\omega(0) = 1 - \Delta$
- Infected (adopting product): $I_\omega(t)$
- Recovered (not adopting product): $R_\omega(t)$

Transmission rule

- Each infected consumer meets other consumers randomly at rate $\beta > 0$
- $S_\omega(t)$, $I_\omega(t)$ and $R_\omega(t)$ are determined dynamically by
  
  \[
  l'_\omega(t) = \beta l_\omega(t) S_\omega(t) p_\omega(t) \\
  R'_\omega(t) = \beta l_\omega(t) S_\omega(t)(1 - p_\omega(t)) \\
  S'_\omega(t) = -\beta l_\omega(t) S_\omega(t) 
  \]

  where $p_\omega(t)$ is the probability of adoption given $\omega$. 
Belief evolution

Upon awareness at \( t \), a consumer’s *interim belief* \( q(t; \emptyset) \) is characterized by

\[
\frac{q(t; \emptyset)}{1 - q(t; \emptyset)} = \frac{\alpha l_g(t)S_g(t)}{(1 - \alpha)l_b(t)S_b(t)}.
\]

Hence, her *posterior beliefs* are

\[
q(t; G) = q(t; \emptyset) \frac{\rho}{\rho q(t; \emptyset) + (1 - \rho)(1 - q(t; \emptyset))},
\]

\[
q(t; B) = q(t; \emptyset) \frac{1 - \rho}{(1 - \rho)q(t; \emptyset) + \rho(1 - q(t; \emptyset))}.
\]
Consumer’s decision

A consumer will

- **Herd on adoption** if $q(t; \emptyset) > \rho$
- **Be sensitive to signals** if $q(t; \emptyset) \in (1 - \rho, \rho)$
- **Herd on non-adoption** if $q(t; \emptyset) < 1 - \rho$

$q(t; \emptyset) = \rho$ or $1 - \rho$: to be discussed later.
Result 1: lifecycle

Unique equilibrium

- $\alpha > \rho$
  - Herd on adoption
  - $p(t) = \alpha$

- $\frac{1}{2} < \alpha < \rho$
  - Herd on adoption
  - $p(t) > \rho$ declining
  - $p(t) = \rho$
  - $p(t) \in (1 - \rho, \rho)$
  - Partial herding
  - Sensitivity to signals
  - $p(t) < 1 - \rho$
  - Product obsolescence

- $1 - \rho < \alpha < \frac{1}{2}$
  - Sensitivity to signals
  - $p(t) = \rho$
  - $p(t) \in (1 - \rho, \rho)$
  - Increasing

- $\alpha < 1 - \rho$
  - Herd on non-adoption
  - $p(t) = \alpha$
Result 1: lifecycle

Two forces on consumer belief

When predecessors herd on adoption

Effect on belief

Elapsed time before awareness

Private information of predecessors

When predecessors use a mixed strategy

Effect on belief

Elapsed time before awareness

Private information of predecessors

When predecessors are sensitive to signals

Effect on belief

Elapsed time before awareness

Private information of predecessors
Result 1: lifecycle

Belief evolution

$p(t)$ [interim belief]

\[
\begin{align*}
\text{HERDING} & : \quad 0 < t < t_1 \\
\text{PARTIAL HERDING} & : \quad t_1 < t < t_2 \\
\text{SIGNAL-SENSITIVE} & : \quad t_2 < t < t_3 \\
\text{OBSOLESCENT} & : \quad t > t_3
\end{align*}
\]

For $\alpha > 1/2$:

- $p(0+) = \rho$
- $p(t) = \frac{1}{2}$ at $t_2$
- $p(t) = 1 - \rho$ for $t > t_3$

For $\alpha < 1/2$:

- $p(0+) = \rho$
- $p(t) = \frac{1}{2}$ at $t_3$
- $p(t) = 1 - \rho$ for $t < t_1$
Implications of consumer equilibrium

With viral social learning:

- True quality remains unrevealed
- Product obsolescence always occurs
- Behavior of product reputation and adoption likelihood are sensitive to initial beliefs
  - High $\alpha \rightarrow$ both are monotone in time
  - Low $\alpha \rightarrow$ neither is monotone in time
Result 2: robust occurrence of viral campaign

A model with endogenous campaign choice

- Nature draws quality $\omega \in \{g, b\}$ with distribution $(\alpha, 1 - \alpha)$
- Producer’s choice (before knowing quality): $t = T$ to stop viral campaign and switch to advertising campaign

Trade off in a viral campaign

- Some adoption given bad signal
- Long viral campaign $\rightarrow$ no adoption in advertising campaign
Result 2: robust occurrence of viral campaign

**Theorem**

*The earliest optimal stopping time $T^*$ is always positive.*

- Belief from advertising campaign is below $\alpha$, and decreases in $T$
- Always better to allow some viral spread
- Two possibilities of exact optimum
  - Stop in the middle $\rightarrow$ consumers with good signal buy from advertising campaign
  - Never stop $\rightarrow$ no one buys from advertising campaign
Result 2: robust occurrence of viral campaign

Numerically, stopping in the middle is almost always optimal.
Result 3: Goldilocks effect

A model with endogenous campaign choice + quality choice

- Firm commits to marketing strategy: begin with viral campaign and switch to advertising campaign at $t = T$
- Firm observes cost $c > 0$ to improve quality
- $c \sim F(c)$ on $[0, \bar{c}]$

If there were no viral campaign, the market equilibrium induces a fraction $\hat{\alpha} = F(2\rho - 1)$ of good quality firm.
Result 3: Goldilocks effect

**Proposition**

Suppose that $\hat{\alpha} \in \left(\frac{1}{2}, \rho\right)$. A market equilibrium always exists, and in every market equilibrium, $\alpha^* \leq \hat{\alpha}$. The inequality is strict if $\hat{\alpha} > \frac{1}{2}$. 

![Graph showing the relationship between $\alpha$ and $\rho$]
Result 3: Goldilocks effect

Proposition

Suppose that $\hat{\alpha} \in (1 - \rho, \frac{1}{2})$. In every market equilibrium with profitable advertising campaign, $\alpha^* > \hat{\alpha}$. 
Several topics for future research:

- Pricing
- Temporary infectiousness
- Option to wait
- Reversible adoption decisions